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BULGARIAN MATH 6

OR 9^B

PROBLEMS COLLECTION AND WORKBOOK

**adapted for U.S. High Schools
used in many public schools abroad**

Translated, Adapted, and Augmented
by

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Founder and Director
of the Berkeley Math Circle

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PART I

9B PROBLEMS COLLECTION

The 9B Problems Collection (PC) contains over 2,100 problems grouped in 6 topics or 38 tests. It will save the teacher time in searching for more problems of various difficulty. The 9B PC can be used for independent work by the student, and to assign homework, to conduct tests, and for practical work with applications by the teacher.

Problems in the 9B PC correspond to each chapter in the 9B Textbook and are grouped in three levels:

100 Level A: For Everyone

Level A problems are for all students. They will strengthen and extend the basic knowledge and skills needed to understand the 9B material.

200 Level B: For Excellent Preparation

Level B problems should be addressed by any student who wishes to master the 9B material and obtain excellent preparation for future math courses in high school and college.

300 Level C: For Math Contests

Level C problems are intended for the strongest students who wish to go beyond the 9B textbook material in order to prepare for math contests or a math major in college. Work on these problems with teacher support and appropriate problem-solving background.

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8. Irrational Expressions, I

LEVEL A: PROBLEMS FOR EVERYONE

Find the allowable values of x in the expressions:

- 1 a) $\sqrt{x+6}$; b) $\sqrt{x-4}$;
c) $\sqrt{2x+3}$; d) $\sqrt{3x-1}$.
- 2 a) $\sqrt{5-x}$; b) $\sqrt{-x+7}$;
c) $\sqrt{-3x+1}$; d) $\sqrt{-5x-2}$.
- 3 a) $\frac{5}{\sqrt{x-1}}$; b) $\frac{x}{\sqrt{x+3}}$;
c) $\frac{3}{\sqrt{2-x}}$; d) $\frac{7}{\sqrt{5-x}}$.
- 4 a) $\sqrt{(x+3)^2}$; b) $\sqrt{x^2+5}$;
c) $\sqrt{-x^2}$; d) $\sqrt{-x^2-9}$.
- 5 a) $\sqrt{|x|}$; b) $\sqrt{-|x|}$;
c) $\sqrt{-|x-3|}$; d) $\frac{1}{\sqrt{|x-5|}}$.

Calculate the value of the expression:

- 6 $\sqrt{x+2}$ for x : a) 2; b) -2; c) -1; d) 7.
- 7 $\sqrt{2x+3}$ for x : a) 3; b) -1; c) $\frac{1}{2}$; d) -1.5.
- 8 $\sqrt{7-x}$ for x : a) -2; b) -9; c) 3; d) 6.

Calculate:

- 9 a) $\sqrt{121 \cdot 16}$; b) $\sqrt{-16 \cdot (-9)}$;
c) $\sqrt{6 \cdot 15 \cdot 10}$; d) $\sqrt{0.36 \cdot 25 \cdot 14 \cdot 22 \cdot 77}$.
- 10 a) $\sqrt{12 \frac{24}{25}}$; b) $\sqrt{\frac{(-3)^2 \cdot 144}{4 \cdot 121}}$;
c) $\sqrt{\frac{(-4)^2 \cdot (-3)^2}{(-6)^2 \cdot (-2)^2}}$; d) $\sqrt{\frac{(-3)^2 \cdot (-13)}{39 \cdot (-12)}}$.

Calculate the products:

- 11 a) $\sqrt{3} \cdot \sqrt{12}$; b) $\sqrt{5} \cdot \sqrt{20}$;
c) $\sqrt{2} \cdot \sqrt{32}$; d) $\sqrt{6} \cdot \sqrt{24}$.
- 12 a) $\sqrt{0.4} \cdot \sqrt{0.1}$; b) $\sqrt{0.03} \cdot \sqrt{3}$;
c) $\sqrt{2.5} \cdot \sqrt{40}$; d) $\sqrt{0.2} \cdot \sqrt{125}$.
- 13 a) $\sqrt{12} \cdot \sqrt{\frac{1}{3}}$; b) $\sqrt{18} \cdot \sqrt{\frac{1}{2}}$;
c) $\sqrt{2} \cdot \sqrt{\frac{2}{9}}$; d) $\sqrt{10} \cdot \sqrt{\frac{2}{5}}$.

Calculate:

- 14 a) $\sqrt{82^2 - 18^2}$; b) $\sqrt{17^2 - 8^2}$;
c) $\sqrt{53^2 - 28^2}$; d) $\sqrt{37^2 - 12^2}$.
- 15 a) $\sqrt{36 \cdot 31 + 36 \cdot 69}$; b) $\sqrt{31 \cdot 49 + 33 \cdot 49}$;
c) $\sqrt{107 \cdot 25 - 26 \cdot 25}$; d) $\sqrt{137 \cdot 64 - 56 \cdot 64}$.
- 16 a) $\sqrt{1,764}$; b) $\sqrt{7,056}$; c) $\sqrt{11,025}$; d) $\sqrt{3,969}$.
- 17 a) $\frac{\sqrt{108}}{\sqrt{3}}$; b) $\frac{\sqrt{98}}{\sqrt{2}}$; c) $\frac{\sqrt{275}}{\sqrt{11}}$; d) $\frac{\sqrt{343}}{\sqrt{7}}$.
- 18 a) $\frac{\sqrt{50} + \sqrt{8}}{\sqrt{2}}$; b) $\frac{\sqrt{75} + \sqrt{12}}{\sqrt{3}}$;
c) $\frac{\sqrt{20} + \sqrt{80}}{\sqrt{5}}$; d) $\frac{\sqrt{112} - \sqrt{63}}{\sqrt{7}}$.
- 19 Move as much as possible out of the radical:
a) $\sqrt{32}$; b) $\sqrt{75}$; c) $\sqrt{28}$; d) $\sqrt{99}$.
- 20 Rationalize the denominator of the fractions:
a) $\sqrt{\frac{3}{7}}$; b) $\sqrt{\frac{5}{3}}$; c) $\sqrt{\frac{6}{5}}$; d) $\sqrt{\frac{7}{2}}$.

Move a factor under the radical:

- 21 a) $3\sqrt{2}$; b) $2\sqrt{7}$; c) $5\sqrt{3}$; d) $4\sqrt{5}$.
- 22 a) $-2\sqrt{3}$; b) $-3\sqrt{5}$; c) $-5\sqrt{2}$; d) $-2\sqrt{7}$.

Compare the numbers:

- 23 a) $2\sqrt{5}$ and $3\sqrt{2}$; b) $7\sqrt{2}$ and $5\sqrt{3}$;
c) $3\sqrt{7}$ and $4\sqrt{5}$; d) $7\sqrt{3}$ and $9\sqrt{2}$.
- 24 a) $-3\sqrt{2}$ and $-2\sqrt{5}$; b) $-4\sqrt{3}$ and $-5\sqrt{2}$;
c) $-6\sqrt{2}$ and $-5\sqrt{3}$; d) $-9\sqrt{2}$ and $-7\sqrt{3}$.

Rewrite the radicals in standard form:

- 25 a) $\sqrt{50}$; b) $\sqrt{27}$; c) $\sqrt{24}$; d) $\sqrt{63}$.
- 26 a) $\sqrt{\frac{3}{5}}$; b) $\sqrt{\frac{7}{2}}$; c) $\sqrt{\frac{9}{2}}$; d) $\sqrt{\frac{4}{7}}$.
- 27 a) $\sqrt{8x^3}, x \geq 0$; b) $\sqrt{50a^3}, a \geq 0$;
c) $\sqrt{\frac{36a^3}{5}}, a \geq 0$; d) $\sqrt{\frac{4a^3}{3}}, a \geq 0$.
- 28 Are the following radicals similar:
a) $\sqrt{12}$ and $\sqrt{27}$; b) $\sqrt{50}$ and $\sqrt{32}$;
c) $\sqrt{98}$ and $\sqrt{8}$; d) $\sqrt{75}$ and $\sqrt{45}$?

Perform the operations to simplify the expressions:

29 a) $\sqrt{8} + \sqrt{18} + \sqrt{50}$; b) $\sqrt{12} + \sqrt{27} + \sqrt{75}$;
c) $\sqrt{20} + \sqrt{45} + \sqrt{80}$; d) $\sqrt{28} + \sqrt{63} - \sqrt{175}$.

30 a) $\sqrt{8} - \sqrt{27} + \sqrt{72} - \sqrt{12}$;
b) $\sqrt{75} + \sqrt{50} - \sqrt{27} - \sqrt{32}$;
c) $\sqrt{45} - \sqrt{8} + 3\sqrt{5} - \sqrt{32}$;
d) $\sqrt{27} + \sqrt{50} - \sqrt{12} - \sqrt{18}$.

31 a) $\frac{\sqrt{75} - 3\sqrt{12} - \sqrt{27}}{\sqrt{3}}$;
b) $\frac{3\sqrt{8} - 2\sqrt{18} + \sqrt{50}}{\sqrt{2}}$;
c) $\frac{\sqrt{45} + 3\sqrt{20} - \sqrt{125}}{\sqrt{5}}$;
d) $\frac{\sqrt{28} - \sqrt{63} + \sqrt{175}}{2\sqrt{7}}$.

32 a) $3(5 - 2\sqrt{2}) - 4(3 - \sqrt{2})$;
b) $\sqrt{2}(\sqrt{3} - 1) + \sqrt{3}(\sqrt{3} - \sqrt{2})$;
c) $(\sqrt{3} + 1)(\sqrt{3} - 2)$;
d) $(2\sqrt{2} - 3)(\sqrt{2} + 3)$.

33 a) $(\sqrt{7} + \sqrt{2})(\sqrt{7} - \sqrt{2})$;
b) $(3 - \sqrt{5})(3 + \sqrt{5})$;
c) $(\sqrt{11} + 3)(\sqrt{11} - 3)$;
d) $(2\sqrt{3} - \sqrt{10})(2\sqrt{3} + \sqrt{10})$.

34 a) $(\sqrt{5} - 2)^2$; b) $(\sqrt{7} + \sqrt{3})^2$;
c) $(2\sqrt{3} - 1)^2$; d) $(\sqrt{3} - \sqrt{5})^2$.

35 a) $\frac{6 - \sqrt{2}}{3} - \frac{4 - \sqrt{3}}{2}$; b) $\frac{4 + \sqrt{3}}{4} - \frac{5 + \sqrt{2}}{5}$;
c) $\frac{2 + 3\sqrt{2}}{4} - \frac{15\sqrt{2} - 3}{6}$;
d) $\left(3\sqrt{\frac{2}{3}} - 2\sqrt{3} - \sqrt{6}\right)\left(2\sqrt{\frac{2}{3}} - 4\sqrt{\frac{3}{2}} + \frac{3}{2}\sqrt{6}\right)$.

36 a) $(\sqrt{2} - \sqrt{3})(1 + \sqrt{2} + \sqrt{3})$;
b) $(2\sqrt{2} - 1)(\sqrt{3} + \sqrt{2} + 1)$;
c) $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2} - 1)$;
d) $(2\sqrt{7} + \sqrt{2})(4\sqrt{7} - 2\sqrt{2} + 1)$.

37 a) $(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2} - 1) + 2\sqrt{3}$;
b) $(3\sqrt{2} - 2\sqrt{3})(\sqrt{2} + \sqrt{3} - 1) - \sqrt{3}(2 - \sqrt{6} - 3\sqrt{3} + \sqrt{2})$;
c) $(2 + \sqrt{7} + \sqrt{11})(\sqrt{7} - \sqrt{11} + 1) - 3\left(3 + \sqrt{7} - \frac{\sqrt{11}}{3}\right)$;
d) $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{3} - 1) - \sqrt{5}(\sqrt{3} - \sqrt{2} - 1) - \sqrt{2}(1 - \sqrt{2} - \sqrt{3})$.

38 a) $(\sqrt{3} - \sqrt{2})^2 - (\sqrt{6} - 1)^2$;
b) $(2\sqrt{2} - \sqrt{3})^2 - (\sqrt{6} - 2)^2$;
c) $(\sqrt{3} - \sqrt{5})(\sqrt{3} + 1) - (\sqrt{5} - 2)^2 + 6 + \sqrt{15}$;
d) $(\sqrt{7} + \sqrt{3})^2 - (\sqrt{7} + 2)(2\sqrt{3} - 1) + 4\sqrt{3}$.

39 a) $\frac{(\sqrt{3} - 1)^2}{2} - \frac{(2 - \sqrt{3})^2}{3}$;
b) $\frac{(\sqrt{3} - \sqrt{2})(\sqrt{2} - 1)}{3} - \frac{(1 - \sqrt{3})^2}{2}$;
c) $\left(\frac{1 - \sqrt{2}}{\sqrt{3}}\right)^2 - \left(\frac{\sqrt{3} - 2}{\sqrt{2}}\right)^2$;
d) $\left(\frac{2\sqrt{2} - 1}{\sqrt{3}}\right)^2 - \left(\frac{2 - \sqrt{3}}{\sqrt{2}}\right)^2$.

Rationalize the denominators:

40 a) $\frac{5}{\sqrt{6}}$; b) $\frac{3}{\sqrt{7}}$; c) $\frac{\sqrt{2}}{\sqrt{3}}$; d) $\frac{7 - \sqrt{2}}{\sqrt{3}}$.

41 a) $\frac{4}{\sqrt{7} - \sqrt{3}}$; b) $\frac{15}{\sqrt{7} - \sqrt{2}}$;
c) $\frac{\sqrt{3}}{\sqrt{5} - \sqrt{3}}$; d) $\frac{12}{2\sqrt{2} - \sqrt{5}}$.

42 a) $\frac{3}{\sqrt{x} - \sqrt{2}}$; b) $\frac{2}{\sqrt{x} - 2\sqrt{2}}$;
c) $\frac{3}{\sqrt{x - 1} - \sqrt{2}}$; d) $\frac{5}{\sqrt{x + 2} - \sqrt{3}}$.

43 a) $\frac{6}{\sqrt{x + 3} - \sqrt{x}}$; b) $\frac{x^2 + 1}{\sqrt{2x^2 + 3} + \sqrt{x^2 + 2}}$;
c) $\frac{3}{\sqrt{x + 4} - \sqrt{x + 1}}$; d) $\frac{2x^2 + 1}{\sqrt{3x^2 + 5} - \sqrt{x^2 + 4}}$.

44 a) $\frac{2}{\sqrt{3} + \sqrt{2} + 1}$; b) $\frac{12}{\sqrt{2} + \sqrt{3} - \sqrt{5}}$;
c) $\frac{12}{2 + \sqrt{6} - \sqrt{10}}$; d) $\frac{2}{\sqrt{5} + \sqrt{2} - \sqrt{3}}$.