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BULGARIAN MATH 5

OR 9A

PROBLEMS COLLECTION AND WORKBOOK

**adapted for U.S. High Schools
used in many public schools abroad**

Translated, Adapted, and Augmented
by

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of the Berkeley Math Circle

ΑΡΧΗ ΜΕΤΑ
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PART I

PROBLEMS COLLECTION

This 9A Problems Collection (PC) contains over 2,250 problems. It will save the teacher time in searching for more problems of various difficulty. The 9A PC can be used for independent work by the student, to assign homework, to conduct tests, and for practical work with applications. Problems in the 9A PC correspond to each chapter in the 9A Textbook, grouped in three levels:

100 Level A: For Everyone

Level A problems are for all students. They will strengthen and extend the basic knowledge and skills needed to understand the 9A material.

200 Level B: For Excellent Preparation

Level B problems should be addressed by any student who wishes to master the 9A material and obtain excellent preparation for future math courses.

300 Level C: For Math Contests

Level C problems are intended for the strongest students who wish to go beyond the 9A textbook material and/or to prepare for math contests. Work on these problems only with the proper support and background.

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These worksheets come directly from the 9A textbook. They consist of the examples from each lesson that have space for students' solutions. The worksheets can be used for directed individual or group work in class.

The worksheets are to be used in tandem with the textbook. In addition to completing these worksheets, the full solutions, discussions, and remarks in the textbook should be carefully read and understood by the student.

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Graph of the Quadratic Function

L9

Example 1 Draw the graph of the function
 $y = x^2 - 4x + 7$.

Solution:

Example 2 Draw the graph of the function
 $y = -x^2 - 2x + 3$.

Solution:

Properties of Quadratic Functions

L10

Example 1 Draw the graphs of the functions:

a) $y = \frac{1}{2}x^2 + 4x + 6$;

Solution:

b) $y = -\frac{1}{2}x^2 + 4x - 1$.

Solution:

Example 1

Solve the system:

$$\begin{cases} x^2 + 2xy - 3y^2 = 0 \\ x^2 + y^2 - 10y = 0. \end{cases}$$

Solution:

Example 2

Solve the homogeneous system:

$$\begin{cases} x^2 + xy - 2y^2 = 0 \\ xy + x - y = 1. \end{cases}$$

Solution:

Example 3

Solve the system:

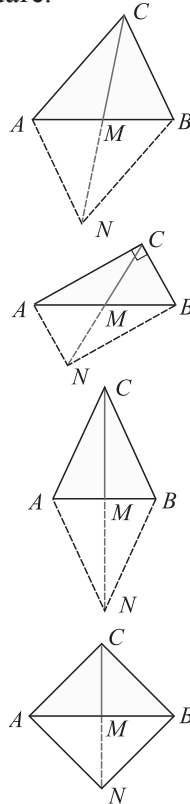
$$\begin{cases} x^2 + xy + y^2 = 7 \\ 4x^2 - xy - 4y^2 = -14. \end{cases}$$

Solution:

Example 2 In $\triangle ABC$, median CM is extended to segment $MN = CM$ ($N \neq C$). If $\triangle ABC$ is ___ then $ANBC$ is ___:

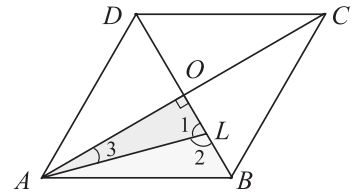
- a) arbitrary $\triangle \Rightarrow$ a parallelogram;
- b) right $\triangle \Rightarrow$ a rectangle.
- c) isosceles $\triangle$ ($AC = BC$) $\Rightarrow$ a rhombus.
- d) right isosceles $\triangle$ ($AC = BC$) $\Rightarrow$ a square.

Proof:



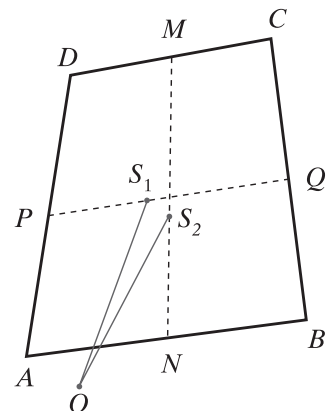
Example 3 A rhombus $ABCD$ has perimeter 84 cm. Find the length of its shorter diagonal if the angle bisector of $\angle BAC$ forms a 75° -angle with BD .

Solution:



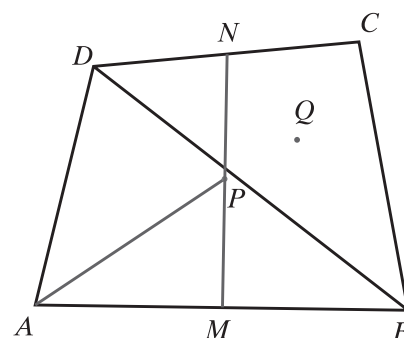
Example 1 Prove that the midpoints of the segments whose endpoints are midpoints of the opposite sides in an arbitrary quadrilateral coincide.

Proof:



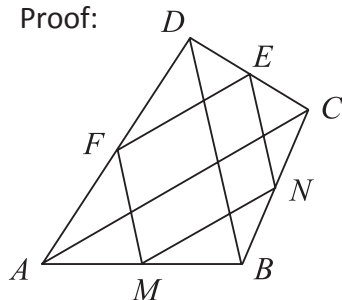
Example 2 In quadrilateral $ABCD$, M and N are the midpoints of sides AB and CD , respectively, and P is the midpoint of MN . Point Q , inside $\triangle BCD$, satisfies the following vector equality: $\vec{AQ} = \frac{1}{3}(\vec{AB} + \vec{AD} + \vec{AC})$. Prove that points A , P , and Q lie on one line.

Proof:



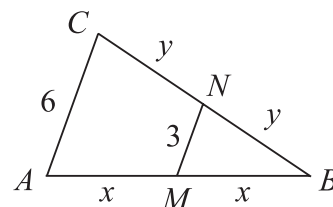
Example 1 Prove that the midpoints of any quadrilateral are the vertices of a parallelogram.

Proof:



Example 2 MN ($M \in AB$, $N \in BC$) is a midsegment in $\triangle ABC$, $AC = 6$ cm, and $P_{AMNC} = 19$ cm. What is $P_{\triangle MNB}$?

Solution:

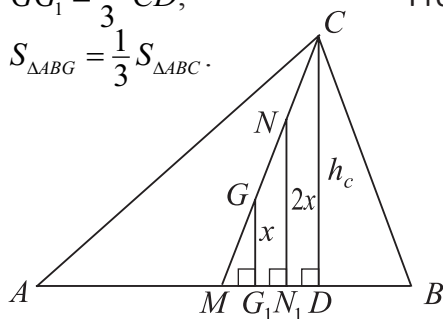


Example 3 In $\triangle ABC$, CD ($D \in AB$) is an altitude and point G is the centroid. If GG_1 ($G_1 \in A$) is the distance from G to AB , show that:

a) $GG_1 = \frac{1}{3} \cdot CD$;

b) $S_{\triangle ABG} = \frac{1}{3} S_{\triangle ABC}$.

Proof:



Example 3 Solve the inequality $\frac{3}{x^2 - 4x + 3} \geq \frac{4}{x^2 - 5x + 4}$.

Solution:



Example 4 Solve the inequality $\frac{x^3 - 4}{x^2 - 4} \leq 1$.

Solution:



Example 5 A motorboat must travel from port A to port B and back in no more than 3 h. What should the motorboat's speed (in calm waters) be if the speed of the current of the river is 5 km/h, the distance between A and B is 28 km, and the boat rests at port B for 40 min?

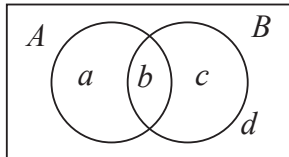
Solution:

	S (km)	V (km/h)	t (h)
with current			
against current			



Example 1

Using the figure, express the given probabilities in terms of a , b , c and d :



a) $P(A) =$

b) $P(C) =$

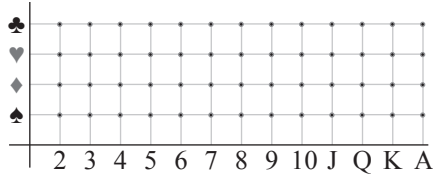
c) $P(A \cap C) =$

d) $P(A \cup C) =$

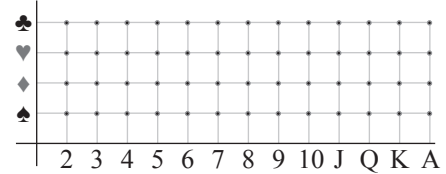
e) $P(A) + P(C) - P(A \cap C) =$

Example 2 We have a deck of 52 cards. Without looking, we draw one of them. Represent graphically the set of all elementary outcomes and find the probability that the card drawn is:

a) 6 or a King;

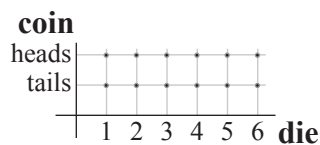


b) 6 or hearts.



Example 3 We simultaneously roll a fair coin and a die. Represent graphically the set of all elementary outcomes and find the probability that the die will show 5 or the coin will show tails.

Solution:



Example 4 We roll two fair dice. Find the probability that at least one die shows 3.

Solution:

die 1

