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BULGARIAN MATH 3

OR **8** A

WORKBOOK

adapted for U.S. High Schools
and Excellent Middle Schools

Translated, Adapted, and Augmented
by

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BULGARIAN MATH 3 (8A) WORKBOOK
for U.S. High Schools and Excellent Middle Schools

Edition in English, 2017:

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 - © Zdravka Krumova Paskaleva, Maya Sabcheva Alashka – authors, 2017
 - © Zvezdelina Entcheva Stankova, Ph. D., Director of Berkeley Math Circle at University of California at Berkeley, <http://mathcircle.berkeley.edu>, Teaching Professor of Mathematics at University of California at Berkeley, CA 94720 – translator, adapter, editor, graphic designer, and partial author of edition in English, 2017.
 - © Berkeley Math Circle Logo used on Cover with permission from Berkeley Math Circle.
 - © Emil Hristov – cover designer, 2017
- Copyeditors: Thomas Rike, math teacher at Oakland High School; Kelli Talaska, instructor at UC Berkeley

ISBN: 978-954-779-260-9

Edition in Bulgarian, 2013:

- © Publishing House “Archimedes 2” EOOD – original edition in Bulgarian, 2013.
- © Zdravka Krumova Paskaleva, Maya Sabcheva Alashka – authors, 2013.
- © Angelina Vladislavova Avramova – graphic designer of original edition in Bulgarian, 2013

Printed by Alliance print – Sofia, Bulgaria

The 8A Workbook will strengthen the basic knowledge and skills needed to master the 8A material. To each topic in the 8A Textbook correspond problems in the 8A Workbook that every student should be able to solve. Some problems have a solution and some have the beginning of a solution. After working on such problems, students should be able to complete the remaining problems on their own.

The 8A Worksheets highlight the basic knowledge in the 8A Textbook; have a teaching component; help students learn to organize and correctly write solutions to problems; and save time for the teacher in searching for more problems. The 8A Worksheets can be used for independent work by the student; to assign homework; to conduct tests; and for practical work with applications.

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PART II. 8A LESSON EXERCISES 100

These worksheets come directly from the 8A textbook. They consist of the examples from each lesson that have space for students' solutions. The worksheets can be used for directed individual or group work in class.

The worksheets are to be used in tandem with the textbook. In addition to completing these worksheets, the full solutions, discussions, and remarks in the textbook should be carefully read and understood by the student.

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Example 1 Calculate the value of the expression in two ways. Write your solutions in a correct, complete, and clear way. How many points out of 10 do you think you will get?

$$A = (x + 5)(x - 2) - (x + 3)(x - 4) \text{ if } x = -2.$$

Solution:

Points: for correctness;
 for completeness;
 for clarity;
 total.

Points: for correctness;
 for completeness;
 for clarity;
 total.

Example 2 The perpendicular bisector of leg BC in $\triangle ABC$ ($CA = CB$) intersects side AC in point Q . If $BC = 22$ cm and the perimeter of $\triangle ABQ$ is 36 cm, find the perimeter of $\triangle ABC$ (in cm). Is the solution below CoCoClear? How many points do you think it will receive? Why?

“Solution”: $22 + c = 36$, $c = 36 - 22 = 14$, $2 \cdot 22 + 14 = 44 + 14 = 58$.

- The answer is correct. So, 2 points for correctness.
- All steps are shown, except for the units (in cm), so 4.5 points for completeness.
- Everything is clearly written. We can follow each step in a logical sequence. So, 3 for clarity.
- It seems this is an almost perfect solution, deserving 9.5 out of 10 points!

Points: for correctness;
 for completeness;
 for clarity;
 total.

Your Solution:

Analysis: How did you assign the points above? Explain.

Pair up with a classmate and grade each other's solutions.

Points: for correctness;
 for completeness;
 for clarity;
 total.

Linear Inequalities with One Unknown

L14

Example 4 Solve the inequalities:

a) $2x - \frac{x+5}{3} \leq \frac{13x+5}{6}$;

b) $\frac{x+5}{2} > 1 - \frac{x+3}{4}$.

Representing Solutions by Intervals and Graphs

L15

Example 1 Solve the inequalities and represent their solutions graphically and via intervals:

a) $3x - 2 > 10$;

b) $5x + 8 < 2x - 10$.

Example 2 Solve the inequalities and represent their solutions graphically and via intervals:

a) $2(2x - 3) \leq 7x + 3$;

b) $3(x + 5) \geq 5(x - 23)$.

Inequalities that Reduce to Linear Ones

L16

Example 1 Write an inequality that is equivalent to:

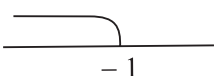
a) $x \in (2, +\infty)$;

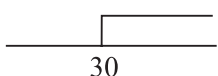
b) $x \in [-3, +\infty)$;

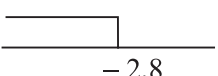
c) $x \in (-\infty, -5)$;

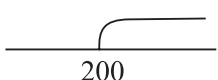
d) $x \in (-\infty, 7]$.

Example 2 If x is a variable, write inequalities whose solutions are represented graphically below:

a)  ;

b)  ;

c)  ;

d) .

L16**Inequalities that Reduce to Linear Ones****Example 3** Solve the inequalities:

a) $\frac{y}{3} + \frac{y-1}{-4} \geq 1$;

b) $\frac{3t}{0.4} + 2 < -\frac{t}{0.2}$.

Example 4 Solve the inequalities:

a) $(x-2)(x^2+1) > 0$;

b) $(x+3)(x^2+4) \leq 0$.

Example 5 Find the values of x for which the expression:

a) $3x - 7$ is larger than 5;

b) $2(x+5)$ is **not** larger than 3;

c) $5 - 2x$ is smaller than 7;

d) $2(3x - 1)$ is **not** smaller than 4.

L17**Inequalities. Exercises****Example 1** Solve the inequalities:

a) $7(x-1) > 7(x+1)$;

b) $4(y-3) - 2y < 2y - 3$;

c) $x(x+2) - (x+1)^2 > -1$.

Example 3 Solve the parametric inequality
 $(x + a)^2 - x(x + 2) > 1$.

Solution:

Inequalities btw Sides and Angles in Triangles

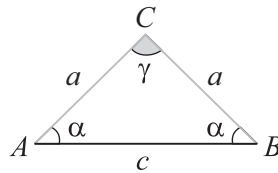
L22

Example 1 Answer the questions:

- Can a triangle be simultaneously equilateral and right? Why?
- Can an obtuse angle in a triangle lie opposite the shortest side? Why?
- Can an acute angle in a triangle lie opposite the longest side? Why?

Example 2 The angle at the vertex of an isosceles triangle is $68^\circ 30'$. Which of the two sides is longer: the base or the leg?

Solution:



Inequalities btw Sides and Angles. Exercises

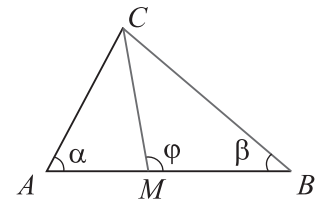
L23

Example 1 A point M is taken on side AB of $\triangle ABC$. Prove that if $BC > AC$ then $CM < BC$.

Given: $\triangle ABC$, $M \in AB$, and $BC > AC$.

Prove: $CM < BC$.

Proof:

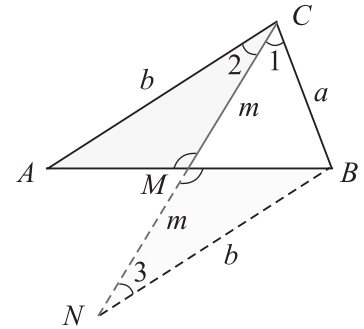


Example 2 Given is $\triangle ABC$ such that $AC > BC$ and CM is a median. Prove that $\angle BCM > \angle ACM$.

Given: $\triangle ABC$, $AC > BC$, and CM a median.

Prove: $\angle BCM > \angle ACM$.

Proof:

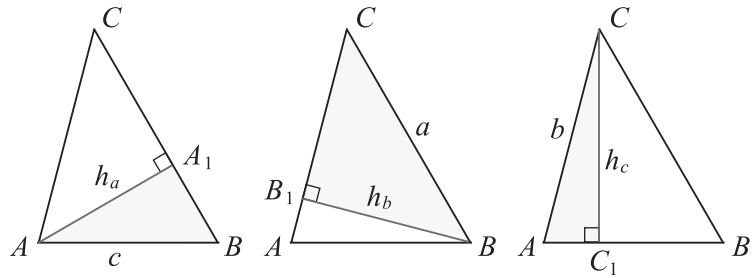


Example 3 Prove that the sum of the altitudes in any triangle is smaller than its perimeter.

Given: $\triangle ABC$, and h_a , h_b , and h_c altitudes.

Prove: $h_a + h_b + h_c < P$.

Proof:

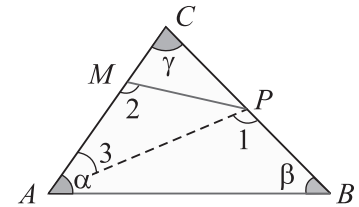


Example 4 In $\triangle ABC$, side AB is the longest. If point P is on side BC and point M on side AC , prove that $MP < AB$.

Given:

Prove:

Proof:

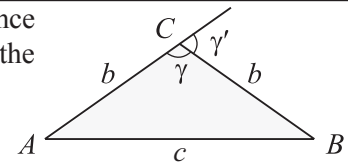


Example 5 The perimeter of an isosceles $\triangle ABC$ ($CA = CB$) is 25 cm, the difference between two of its sides is 4 cm, and the exterior angle at vertex C is acute. Find the sides of the triangle.

Given:

Prove:

Proof:

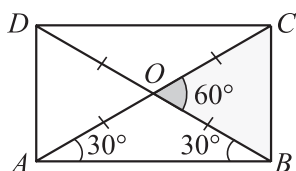


L34

Rectangle

Example 1 Prove that if the angle between the diagonals of a rectangle is 60° then:

- a) this is an angle in an equilateral triangle formed by the diagonals and a side of the rectangle;

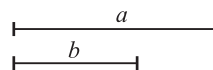


- b) each diagonal makes an angle of 30° with the longer side;

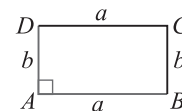
- c) each diagonal is twice as long as the shorter side of the rectangle.

Example 2 Construct a rectangle by two sides.

Given:



Construct: Rectangle $ABCD$.



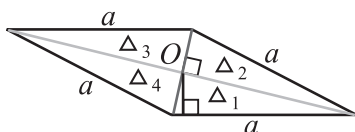
L35

Rhombus

Example 1 Prove that:

- a) the diagonals in a rhombus divide it into four congruent right triangles;
- b) the intersection point O of the diagonals of a rhombus is at equal distances from its sides.

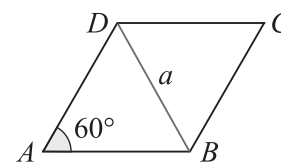
Proof:



Example 2 Given is a rhombus with side a .

Prove that $BD = a \Leftrightarrow \angle A = 60^\circ$.

Proof:



1 Solve the inequalities:

a) $\frac{x}{-3} + \frac{x-2}{2} > 1$;

b) $\frac{x}{0.3} - \frac{x}{0.2} > 5$.

2 Solve the inequalities:

a) $(x-3)(x^2+5) > 0$;

b) $(x-4)(x^2+1) < 0$.

3 First factor and then solve the inequalities:

a) $x^3 + x^2 + 3x + 3 \geq 0$;

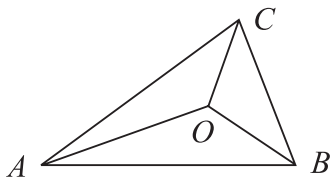
b) $x^3 - 2x^2 + 5x - 10 \leq 0$.

4 Find the values of x for which:

a) the expression $\frac{3x-1}{2}$ is not larger than 4;

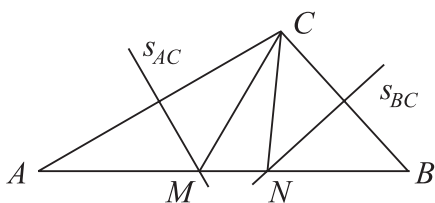
b) the expression $\frac{2x-7}{-3}$ is not smaller than 3.

- 1 In $\triangle ABC$ $\alpha = 40^\circ$ and $\beta = 60^\circ$, and the angle bisectors of $\angle BAC$ and $\angle ABC$ intersect in point O . Arrange in increasing order the lengths of the sides of $\triangle AOB$, $\triangle BOC$, and $\triangle AOC$.

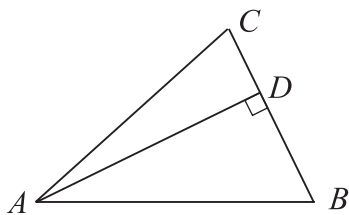


- 2 In $\triangle ABC$ $\alpha = 30^\circ$ and $\beta = 50^\circ$. The perpendicular bisectors of AC and BC intersect AB correspondingly in points M and N . Compare:

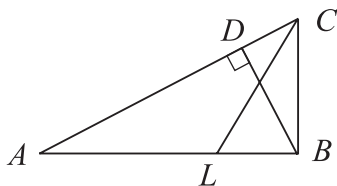
a) the sides of $\triangle MNC$; b) the segments AM , MN , and NB .



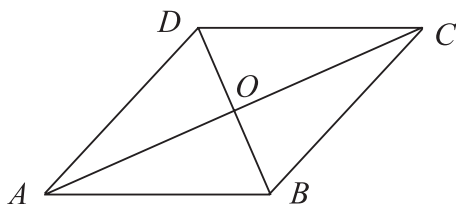
- 3 $\triangle ABC$ is acute and AD is an altitude. Prove that $AD < \frac{1}{2}(AB + AC)$.



- 4 In $\triangle ABC$ $\alpha = 30^\circ$ and $\beta = 90^\circ$, CL is an angle bisector and BD is an altitude. Compare the lengths of the segments CL and BD .

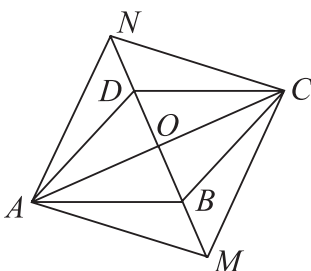


- 1 The diagonals of a rhombus $ABCD$ intersect in point O and $AB = 2OB$. Find the angles of the rhombus.



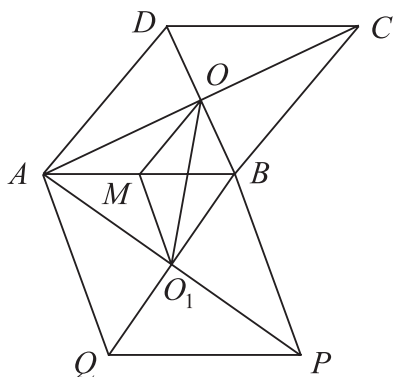
Solution:

- 2 The diagonals of a rhombus $ABCD$ intersect in point O . We have drawn points M and N so that B is the midpoint of OM and D is the midpoint of ON . Prove that $AMCN$ is a rhombus.



Proof:

- 3 In the figure we see a rhombus $ABCD$ with $\angle BAD = 50^\circ$ and a rhombus $ABPQ$ with $\angle BAQ = 70^\circ$. If M is the midpoint of AB , find the angles of $\triangle MOO_1$ where $AC \times BD = O$ and $AP \times BQ = O_1$.



Solution:
