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BULGARIAN MATH 2

OR 7 B

WORKBOOK

adapted for U.S. High Schools
and Excellent Middle Schools

Translated, Adapted, and Augmented
by

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BULGARIAN MATH 1 (7A) WORKBOOK
for U.S. High Schools and Excellent Middle Schools

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The 7B Workbook will strengthen the basic knowledge and skills needed to master the 7B material. To each topic in the 7B Textbook correspond problems in the 7B Workbook that every student should be able to solve. Some problems have a solution and some have the beginning of a solution. After working on such problems, students should be able to complete on their own the remaining problems.

The 7B Worksheets highlight the basic knowledge in 7B Textbook; have a teaching component; help students learn to organize and correctly write solutions to problems; and save time for the teacher in searching for more problems. The 7B Worksheets can be used for independent work by the student; to assign homework; to conduct tests; and for practical work with applications.

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PART II. 7B LESSON EXERCISES

These worksheets come directly from the 7B textbook. They consist of the examples from each lesson that have space for students' solutions. The worksheets can be used for directed individual or group work in class.

The worksheets are to be used in tandem with the textbook. In addition to completing these worksheets, the full solutions, discussions, and remarks in the textbook should be carefully read and understood by the student.

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Example 1 Find $\alpha + \beta$ and $\alpha - \beta$ if:
 a) $\alpha = 28^\circ 43'$, $\beta = 20^\circ 58'$;
 b) $\alpha = 90^\circ$, $\beta = 35^\circ 15'$.

Solution:

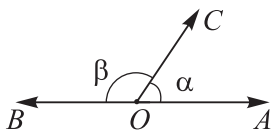
Example 2 The angle bisector of $\alpha = 121^\circ 13'$ divides it into two angles. Find their measures.
 Solution:

Example 1 Angles α and β are supplementary. Find angle β if:
 a) $\alpha = 30^\circ$; b) $\alpha = 47^\circ 30'$; c) $\alpha = 32^\circ 18' 10''$.

Solution:

Example 2 What is the degree measure of an angle that is four times as large as its supplement?

Solution:



Example 2 Noting the theorems used, solve the equation $7x - 2 = x - 5$ and perform a check.
 Solution: Check:

Example 3 Solve the equation
 $5x - 3 - 3x - 1 = 4x - 2 \cdot 7$.
 Solution:

Example 4 Prove that the equations are equivalent:
 (1) $3x - 2 = 5x + 4$ and (2) $7x - 1 = 2x - 16$.
 Proof:

Example 1 Solve the equations:
 a) $2x + 10 = 0$; b) $9x + 3 = 2x + 5$; c) $4.3x + 2 = 2x + 1 \div 0.5$.
 Solution:

Example 2 Solve the equation $6x^2 - (2x - 1)(3x + 1) = 3(1 - x)$.

Solution:

Example 3 Solve the equation $(2x + 1)^2 - 3 = 4(x^2 + 1) + x$.

Solution:

Example 4 Find a number with the property: “The difference of the square of a number and double the number is equal to the square of the difference of this number with the number 1.”

Solution:

Example 1 Solve the equations:

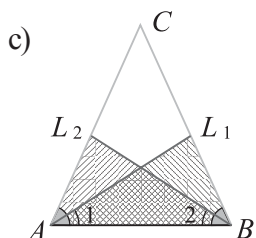
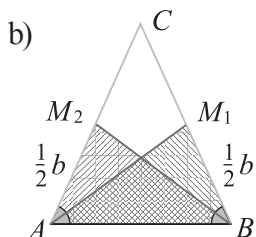
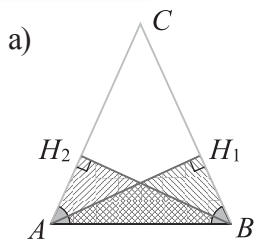
a) $7 - (x - 3)(x + 3) = 2x - (x + 2)^2$; b) $-7x - [2 - 3(x - 1)] + 2x = 5$.

Solution:

Example 1 Given is an isosceles $\triangle ABC$ ($AC = BC$). Prove that:

- a) If AH_1 and BH_2 are altitudes, then $\triangle ABH_1 \cong \triangle BAH_2$ and $AH_1 = BH_2$.
 b) If AM_1 and BM_2 are medians, then $\triangle ABM_1 \cong \triangle BAM_2$ and $AM_1 = BM_2$.
 c) If AL_1 and BL_2 are angle bisectors, then $\triangle ABL_1 \cong \triangle BAL_2$ and $AL_1 = BL_2$.

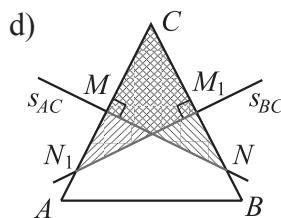
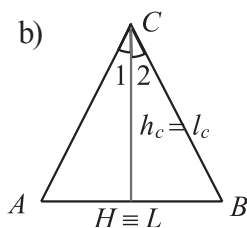
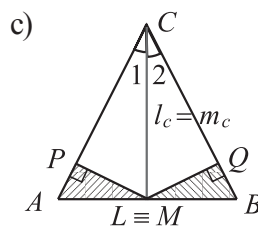
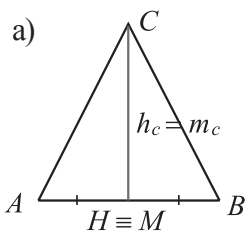
Proof:



Example 3 Prove that if in $\triangle ABC$:

- a) $h_c = m_c$, then $\triangle ABC$ is isosceles with top vertex C .
 b) $h_c = l_c$, then $\triangle ABC$ is isosceles with top vertex C .
 c) $m_c = l_c$, then $\triangle ABC$ is isosceles with top vertex C .
 d) s_{AC} and s_{BC} are perp. bisectors and $MN = M_1N_1$ as in the figure, then $\triangle ABC$ is isosceles with top vertex C .

Proof:



1 Angles α and β are supplementary. Find angle α if:

a) $\beta = 43^\circ$;

b) $\beta = 101^\circ 20'$;

c) $\beta = 20^\circ 20' 20''$.

Solution: a) $\alpha + \beta = 180^\circ$

b)

c)

$$\alpha + 43^\circ = 180^\circ$$

$$\alpha =$$

$$\alpha =$$

2 Angles α and β are supplementary. Find angle β if $\alpha = \beta + 40^\circ$.

Solution:

$$\alpha + \beta = 180^\circ$$

3 Angles α and β are supplementary. How many degrees is each of them if:

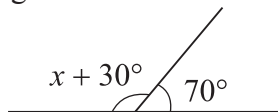
a) $\alpha : \beta = 2 : 1$;

b) $\alpha : \beta = 4 : 5$.

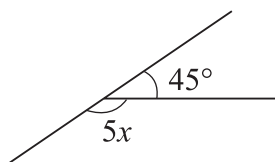
$$\alpha = 2x, \beta =$$

4 Using the notation in the drawing, find x .

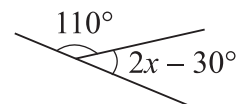
a)



b)



c)



1 Solve the equations:

a) $(x + 2)^2 - x(x + 5) = 2(x - 3)$;

b) $(2x - 1)^2 - (x - 3)^2 = 3x(x + 2)$.

2 Show that the equation
 $(2x + 3)(2x - 3) - (2x - 1)(2x + 1) = 5$
has no solution.

3 Show that the equation
 $(3x - 2)^2 - 8x(x - 2) = (-x - 2)^2$
has any number as a root.

4 Solve the equations:

a) $\frac{x}{3} + 2 = \frac{x}{6} + 1$;

b) $\frac{x-2}{6} = \frac{2x-5}{12}$.

1

Given:

$$\triangle ABC \text{ (} CA = CB \text{)}$$

$$\angle ACB = 120^\circ$$

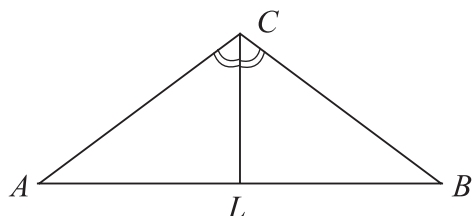
$$CL = 8 \text{ cm}$$

CL angle bisector

Find:

$$AC.$$

Solution:



2

Given:

$$\triangle ABC \text{ (} CA = CB \text{)}$$

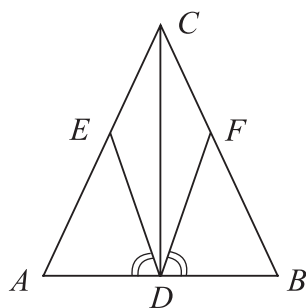
CD altitude

$$\angle ADE = \angle BDF$$

Prove:

$$CD \perp EF.$$

Proof:



3

Given:

$$\triangle ABC \text{ (} CA = CB \text{)}$$

M and N midpoints of CA and CB

Prove:

$$MN \parallel AB.$$

Proof:

