

Zdravka Paskaleva, Maya Alashka

BULGARIAN MATH 1 **OR 7 A** **WORKBOOK**

adapted for U.S. High Schools
and Excellent Middle Schools

Translated, Adapted, and Augmented
by

Zvezdelina Stankova

Founder and Director
of the Berkeley Math Circle

ΑΡΧΗ(Μ)ΕΔ
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BULGARIAN MATH 1 (7A) WORKBOOK
for U.S. High Schools and Excellent Middle Schools

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The 7A Workbook will strengthen the basic knowledge and skills needed to master the 7A material. To each topic in the 7A Textbook correspond problems in the 7A Workbook that every student should be able to solve. Some problems have a solution and some have the beginning of a solution. After working on such problems, students should be able to complete on their own the remaining problems.

The 7A Worksheets highlight the basic knowledge in 7A Textbook; have a teaching component; help students learn to organize and correctly write solutions to problems; and save time for the teacher in searching for more problems. The 7A Worksheets can be used for independent work by the student; to assign homework; to conduct tests; and for practical work with applications.

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Regular (mandatory) problem



Harder problem for excellent proficiency

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The worksheets are to be used in tandem with the textbook. In addition to completing these worksheets, the full solutions, discussions, and remarks in the textbook should be carefully read and understood by the student.

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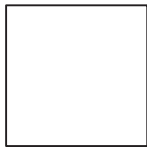
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Example 3

The base edge of a regular quadrilateral prism is 6 cm, and its lateral edge is 10 cm. Find the perimeter and the area:



b

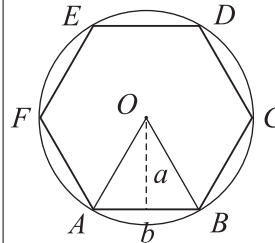
- of the base;
- of a lateral face.

Given:

Solution:

Example 4

A regular hexagonal prism has base edge 8 cm and apothem 6.9 cm. Find the sum of the areas of the two bases.



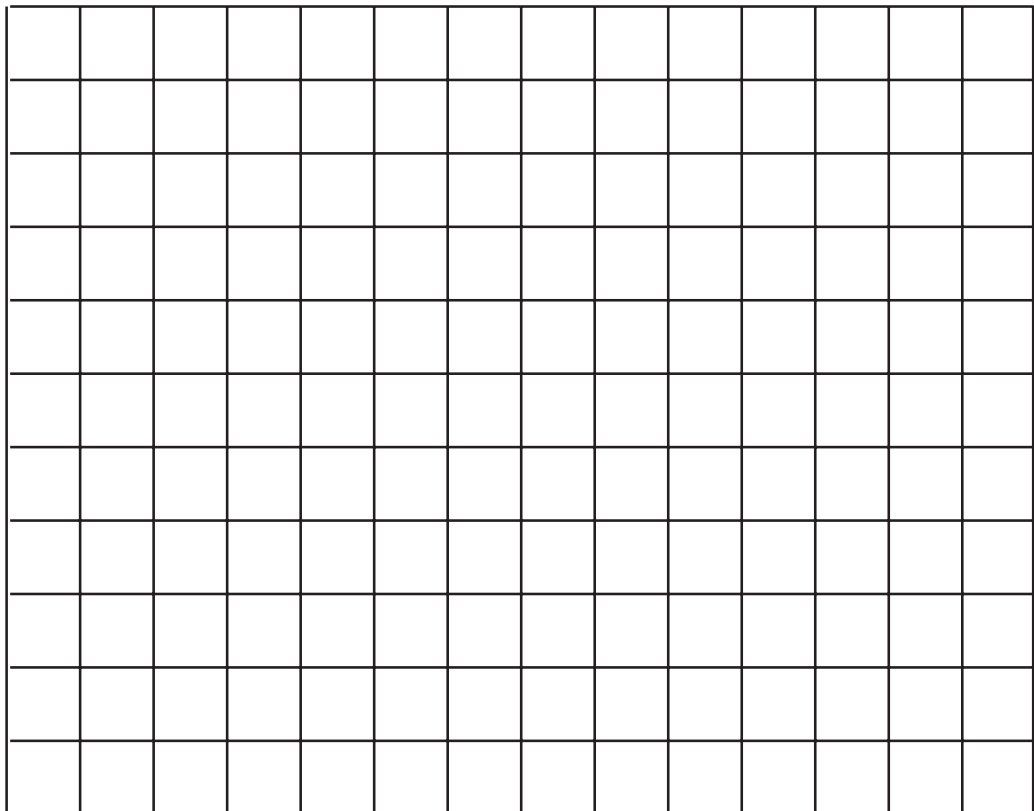
Given:

Solution:

Example 1

On a square grid draw the net of a right triangular prism with height 4.5 cm whose base is a right triangle with legs 3 cm and 4 cm and hypotenuse 5 cm.

Solution: The unit square in the grid has length 1 cm.



Example 2

Find the surface area of the right triangular prism in *Example 1*.

Picture:

Given:

Solution:

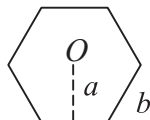
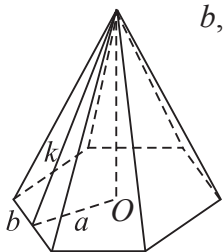
Volume and Surface Area of a Regular Pyramid. Exercises

Example 4

A regular hexagonal pyramid has $S = 69 \text{ cm}^2$, $V = 55.2 \text{ cm}^3$, $k = 5 \text{ cm}$, and $a = 4 \text{ cm}$. Find b , h , and S_1 .

Given:

Solution:



Summary of the Topic “Polyhedral Solids”

Example 1

Given is a regular n -gon prism with base edge b , apothem of the base a , height h , perimeter of the base P , area of the base B , lateral surface area S , surface area S_1 , and volume V . Fill in the table:

Calculations:

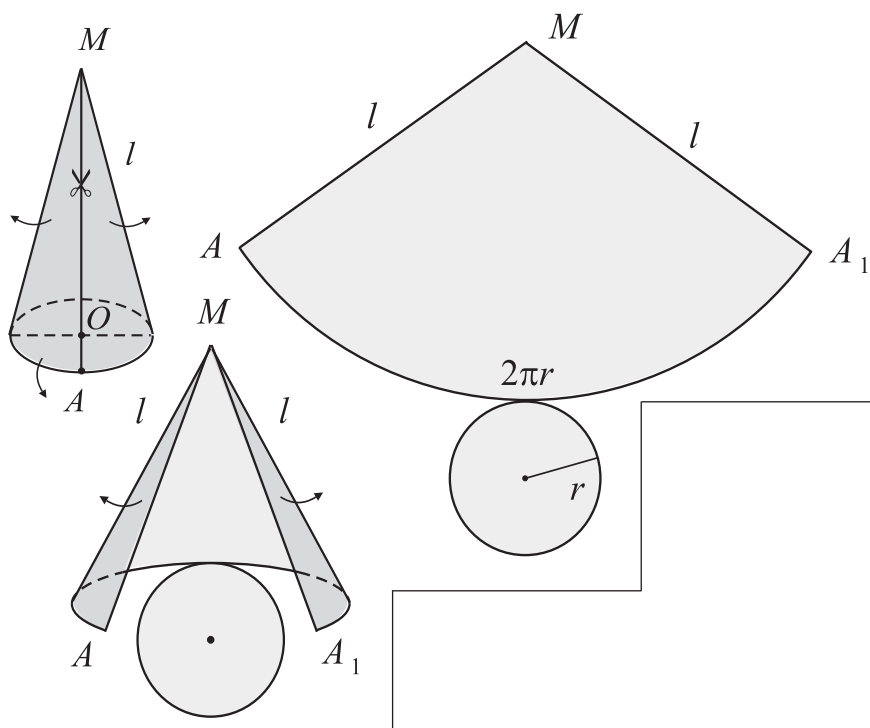
	n	b (cm)	a (cm)	P (cm)	B (cm ²)	h (cm)	S (cm ²)	S_1 (cm ²)	V (cm ³)
a)	5	11.6	8			10			
b)	6		5		87			313.2	
c)	12		8	62.4					499.2

Example 2

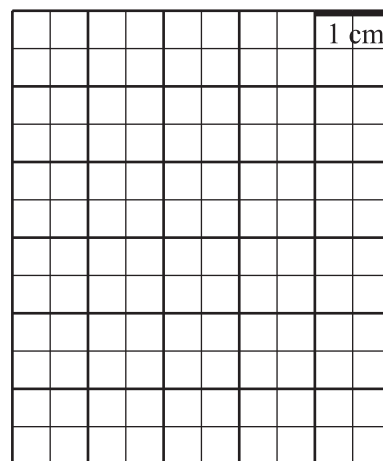
Given is a regular n -gon pyramid with base edge b and apothem of the base a . The pyramid has altitude h , slant height k , perimeter of the base P , area of the base B , lateral surface area S , surface area S_1 , and volume V . Fill in the table:

Calculations:

	n	b (cm)	a (cm)	P (cm)	B (cm ²)	h (cm)	k (cm)	S (cm ²)	S_1 (cm ²)	V (cm ³)
a)	5	11.6	8			6	10			
b)	6		5		87	12			313.2	
c)	12		8	62.4				312		499.2



Example 1 On a square grid (1 unit = 1 cm) draw a cone with radius $r = 1.5$ cm and altitude $h = 4$ cm.

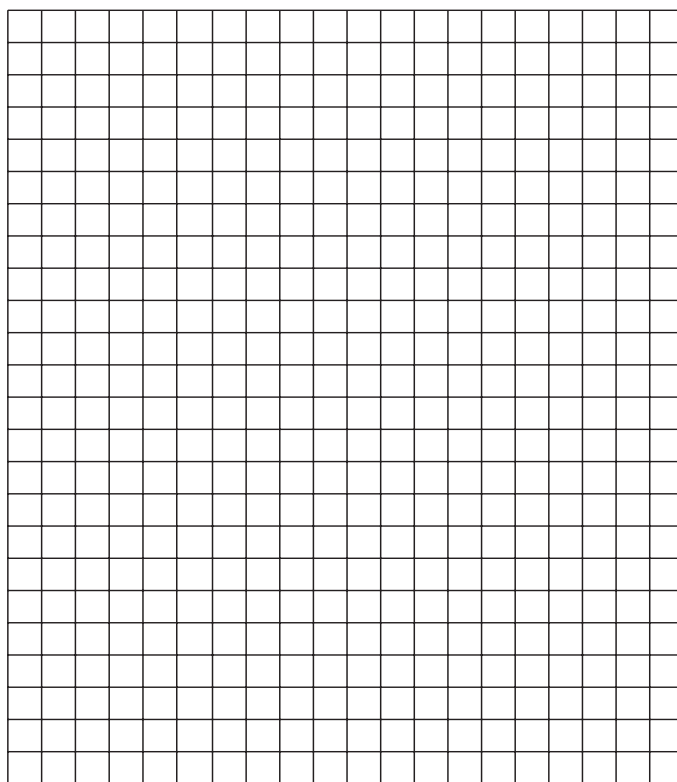
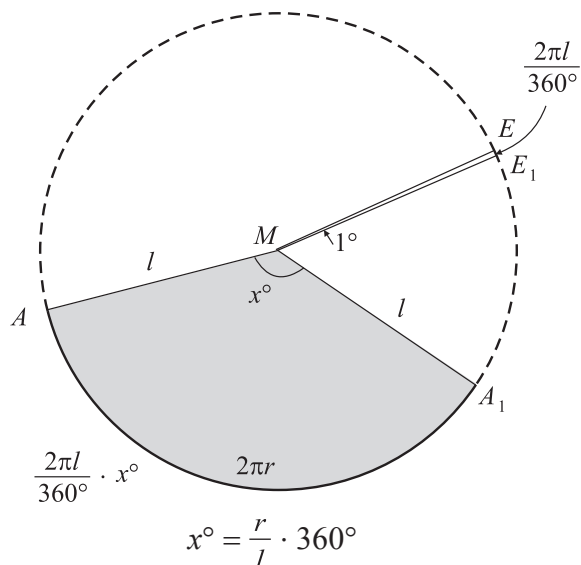


Surface Area of a Cone

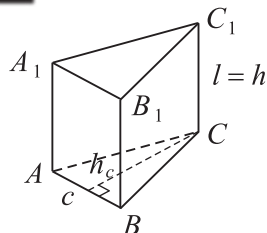
L27

Example 1 Draw the net of a cone of radius $r = 2$ cm and generatrix $l = 5$ cm.

Solution:



1



Given:
 right triangular
 prism
 base: $\triangle ABC$
 $CA = CB = 5 \text{ cm}$
 $AB = 6 \text{ cm}$
 $h_c = 4 \text{ cm}$
 $l = h = 10 \text{ cm}$

 $S = ? \quad S_1 = ?$

Solution:

$$S = P \cdot h \qquad S_1 = S + 2 \cdot B$$

$$P = a + b + c \qquad B = \frac{c \cdot h_c}{2}$$

$$P = 5 + 5 + 6 = 16$$

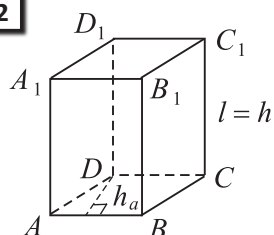
$$P = 16 \text{ cm} \qquad B = \frac{6 \cdot 4}{2}$$

$$S = 16 \cdot 10 \qquad B = 12 \text{ cm}^2$$

$$S = 160 \text{ cm}^2 \qquad S_1 = 160 + 2 \cdot 12$$

$$S_1 = 184 \text{ cm}^2$$

2

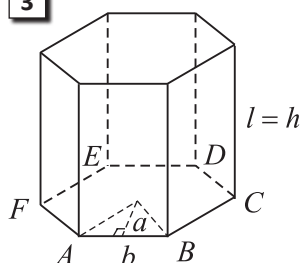


Given:
 right quadrilateral
 prism
 base: rhombus $ABCD$
 $AB = 8 \text{ cm}$
 $h_a = 5 \text{ cm}$
 $l = h = 11 \text{ cm}$

 $S = ? \quad S_1 = ?$

Solution:

3

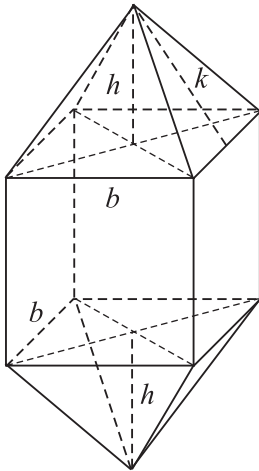


Given:
 right hexagonal
 prism
 base: $ABCDEF$
 $AB = b = 6 \text{ cm}$
 $a = 5.2 \text{ cm}$
 $l = h = 15 \text{ cm}$

 $S = ? \quad S_1 = ?$

Solution:

- 1** A solid is made of a cube and two regular quadrilateral pyramids, as in the drawing. The cube has dimension $b = 6$ cm. Each of the pyramids has slant height 5 cm and height 4 cm. Find the volume and the surface area of the solid.

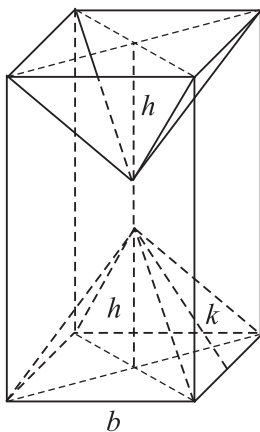


Solution:

$$V_{\text{solid}} = V_{\text{cube}} + 2 \cdot V_{\text{pyramid}}$$

$$S_{\text{solid}} = S_{\text{cube}} + 2 \cdot S_{\text{pyramid}}$$

- 2** From a regular quadrilateral prism two regular quadrilateral pyramids have been carved out as in the drawing. The prism has base edge 10 cm and lateral edge 30 cm. Each of the pyramids has height 12 cm and slant height 13 cm. Find the volume and the surface area of resulting solid.

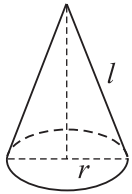


Solution:

$$V_{\text{solid}} = V_{\text{prism}} - 2 \cdot V_{\text{pyramid}}$$

$$S_{\text{solid}} = S_{\text{prism}} + 2 \cdot S_{\text{pyramid}}$$

- 1** A right circular cone has lateral surface area $S = 20\pi \text{ cm}^2$ and slant height $l = 5 \text{ cm}$. Find S_1 (using π).



Given:

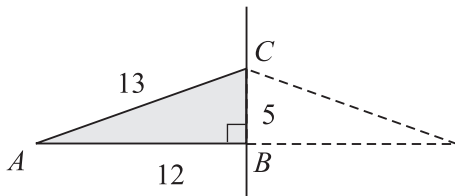
Solution:

$$S_1 = ?$$

- 2** A right $\triangle ABC$ with legs $AB = 12 \text{ cm}$ and hypotenuse $AC = 13 \text{ cm}$ is revolved:
 a) about axis BC ; b) about axis AB .
 Find S_1 of the resulting cone.

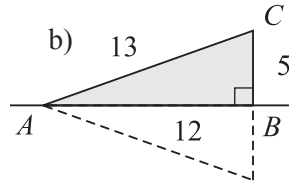
a)

$$r = AB, l = AC$$



b)

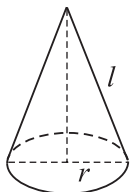
$$r = BC, l = AC$$



- 3** Given is a right circular cone. Complete the table*:

	$r \text{ (cm)}$	$d \text{ (cm)}$	$P \text{ (cm)}$	$B \text{ (cm}^2\text{)}$	$l \text{ (cm)}$	$S \text{ (cm}^2\text{)}$	$S_1 \text{ (cm}^2\text{)}$
1	2				5		
2		8				40π	
3			14π				91π
4					9	45π	

One way to complete row 4 is as follows:



$$S = \pi \cdot r \cdot l$$

$$d = 2 \cdot r$$

$$P = 2\pi r$$

$$B = \pi \cdot r^2$$

$$S_1 = S + B$$

$$45\pi = \pi \cdot r \cdot 9$$

$$d = 10$$

$$P = 10\pi$$

$$B = 25\pi$$

$$S_1 = 45\pi + 25\pi$$

$$r = 5$$

$$S_1 = 70\pi$$

* Perform the calculations on an extra sheet.