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MATHEMATICS

WORKBOOK

6^B
GRADE

Translated and Adapted
by

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The 6B Workbook will strengthen the basic knowledge and skills needed to master the 6B material. To each topic in the 6B Textbook correspond problems in the 6B Workbook that every student should be able to solve. Some problems have a solution and some have the beginning of a solution. After working on such problems, students should be able to complete on their own the remaining problems.

The 6B Worksheets highlight the basic knowledge in 6B Textbook; have a teaching component; help students learn to organize and correctly write solutions to problems; and save time for the teacher in searching for more problems. The 6B Worksheets can be used for independent work by the student; to assign homework; to conduct tests; and for practical work with applications.

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Regular (mandatory) problem



Harder (elective) problem for excellent proficiency and math contests

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These worksheets come directly from the 6B textbook. They consist of the examples from each lesson that have space for students' solutions. The worksheets can be used for directed individual or group work in class.

The worksheets are to be used in tandem with the textbook. In addition to completing these worksheets, the full solutions, discussions, and remarks in the textbook should be carefully read and understood by the student.

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L91**Proper and Improper Fractions**

Example 1 Determine whether the following common fractions are proper or improper:

$$\frac{12}{13}, \frac{10}{10}, \frac{0}{121}, \frac{2}{3}, \frac{7}{8}, \frac{8}{7}, \frac{6}{91}, \frac{102}{8}, \frac{20}{1}, \frac{5}{6}.$$

Solution:

Example 2 Write down all proper fractions with denominator 9.

Solution:

Example 3 Given are the common fractions $\frac{5}{7}, \frac{8}{3}, \frac{1}{5}, \frac{7}{1}$. Switch the places of the numerator and the denominator of each of them and read the resulting fractions.

Solution:

Example 4 Write down the reciprocal fractions of the fractions $\frac{7}{11}, \frac{13}{20}, \frac{0}{9}, \frac{11}{5}, \frac{17}{1}, 21$.

Solution:

L92**Mixed Numbers**

Example 1 Convert the improper fractions into mixed numbers:

a) $\frac{29}{6} =$

b) $\frac{43}{13} =$

Example 2 Convert the improper fractions into mixed numbers:

a) $\frac{67}{5} =$

b) $\frac{102}{13} =$

Example 3 Convert the mixed numbers into improper fractions:

a) $8\frac{3}{7} =$

b) $10\frac{12}{13} =$

Example 4 We are given the fractions $\frac{5}{8}, \frac{17}{6}$, and $5\frac{3}{4}$.
b) Find the reciprocal numbers of the given numbers:

a) Convert the improper fraction into a mixed number:

c) Convert the mixed number into an improper fraction:

Basic Property of Fractions: Expanding

L93

Example 1 Expand the fractions by the number 2: a) $\frac{5}{6} =$ b) $\frac{11}{15} =$ c) $\frac{20}{17} =$

Example 2 Expand the fractions: a) $\frac{7}{12}$ by 2; b) $\frac{3}{2}$ by 12; c) $\frac{1}{4}$ by 6.

a)

b)

c)

Example 3 Find the extra factors of the fraction $\frac{7}{8}$ if:

a) $\frac{7}{8} = \frac{14}{16}$

b) $\frac{7}{8} = \frac{35}{40}$

c) $\frac{7}{8} = \frac{84}{96}$

Example 4 Expand the fractional part of the mixed number $13\frac{2}{3}$ by:

a) 2:

b) 3:

c) 4:

d) 5:

Basic Property of Fractions: Reducing

L94

Example 1 Reduce the fractions $\frac{4}{6}$, $\frac{6}{9}$, $\frac{15}{20}$, $\frac{100}{300}$.

Solution:

Example 2 Reduce the fraction $\frac{24}{42}$.

Solution:

Method I:

Method II:

Example 3 Reduce the fractions: $\frac{40}{50}$, $\frac{300}{800}$, $\frac{220}{550}$, and $\frac{800}{1200}$.

Solution:

Example 4 Reduce the fraction $\frac{66}{2,310}$ by factoring the numerator and the denominator, on the side, into prime factors.

Solution:

Example 5 Reduce the fractions:

a) $\frac{25}{625} =$

b) $\frac{130}{26} =$

c) $\frac{21,000}{105,000} =$

d) $\frac{2,940}{10,010} =$

Example 1 We are shopping in a store. a • The price of the purchase without tax* is \$135. • The tax is 20%. How much is the tax?	Solution:
b • The tax is 20%. • The tax is \$27. What is the price of the purchase without tax?	
c • The price of the purchase without tax is \$135. • The tax is \$27. What percent is the tax?	
Example 2 Find: a) 9.6% of 500 =	b) 135% of 20 =
Example 3 A company hired an accountant with a base monthly salary of \$3,600. For good work, they gave him a bonus: they increased his salary by 15%. a) How much was the bonus? Solution:	b) How much is the new salary?

1 Among the fractions $\frac{2}{3}, \frac{3}{5}, \frac{5}{2}, \frac{7}{3}, \frac{7}{13}, \frac{8}{5}, \frac{5}{9}, \frac{13}{7}, \frac{15}{16}, \frac{21}{43}, \frac{100}{101}$

a) the proper ones are _____

b) the improper ones are _____

2 a) List all proper fractions with denominator 11. _____

b) List all improper fractions with numerator 11. _____

3 Convert the fractions into mixed numbers:

a) $\frac{97}{6} = 97 \div 6 = 16 + \frac{1}{6} = 16\frac{1}{6}$ b) $\frac{55}{3} =$ c) $\frac{73}{8} =$

$$\begin{array}{r} 97 \\ -6 \\ \hline 37 \\ -36 \\ \hline 1 \end{array}$$

4 Complete the table:

Improper fraction	$\frac{4}{3}$	$\frac{5}{4}$	$\frac{6}{5}$	$\frac{7}{6}$	$\frac{7}{3}$	$\frac{8}{7}$	$\frac{8}{3}$	$\frac{9}{4}$	$\frac{11}{4}$	$\frac{19}{7}$
Mixed number	$1\frac{1}{3}$									

5 Convert the mixed numbers into improper fractions:

a) $13\frac{5}{6} = \frac{13 \cdot 6 + 5}{6} = \frac{83}{6}$ b) $21\frac{7}{8} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$ c) $35\frac{2}{5} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

6 Complete the table:

Mixed number	$1\frac{1}{2}$	$1\frac{2}{3}$	$1\frac{5}{6}$	$2\frac{1}{2}$	$2\frac{1}{3}$	$2\frac{4}{5}$	$3\frac{1}{3}$	$4\frac{1}{4}$	$5\frac{2}{3}$	$11\frac{1}{2}$
Improper fraction	$\frac{3}{2}$									

7 Complete the table:

Number	5	$\frac{1}{3}$	$\frac{2}{7}$	$\frac{9}{5}$	13	$\frac{1}{21}$	$\frac{37}{38}$	1	$\frac{51}{63}$	$\frac{102}{97}$
Reciprocal number										

1 Complete the table:

a)

Fractions	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{7}$	$\frac{5}{9}$	$\frac{7}{11}$	$\frac{8}{13}$	$\frac{5}{14}$	$\frac{7}{15}$	$\frac{9}{20}$
Expand by 2	$\frac{2}{4}$									
Expand by 5	$\frac{5}{10}$									

b)

Fractions	$\frac{2}{4}$	$\frac{2}{6}$	$\frac{2}{8}$	$\frac{2}{10}$	$\frac{4}{14}$	$\frac{6}{10}$	$\frac{12}{18}$	$\frac{20}{30}$	$\frac{26}{50}$	$\frac{102}{130}$
Reduce by 2	$\frac{1}{2}$									

2 Reduce the fractions as in the example:

a) $\frac{42}{252} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{3}} \cdot \cancel{7}}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{3}} \cdot \cancel{7}} = \frac{1}{6}$ $\frac{42}{315} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$ $\frac{99}{220} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

b) $\frac{\overset{5}{\cancel{10}} \cdot \cancel{20}}{\underset{12}{\cancel{48}} \cdot \underset{12}{\cancel{24}}} = \frac{5}{12}$ $\frac{20}{45} = \frac{\quad}{\quad}$ $\frac{120}{440} = \frac{\quad}{\quad}$ $\frac{125}{610} = \frac{\quad}{\quad}$ $\frac{84}{294} = \frac{\quad}{\quad}$

3 Complete the table:

Mixed number	$3\frac{1}{2}$	$4\frac{1}{3}$	$15\frac{2}{3}$	$17\frac{7}{8}$	$21\frac{5}{11}$	$28\frac{11}{12}$	$35\frac{13}{14}$	$104\frac{1}{15}$
Expand the fractional part by 3	$3\frac{3}{6}$							

4 Expand the fractions to the same denominators, as in the example:

a) $\frac{7}{8}, \frac{3}{4} \rightarrow \frac{7}{8}, \frac{3}{4} = \frac{3 \cdot 2}{4 \cdot 2} = \frac{6}{8};$ d) $\frac{11}{14}, \frac{1}{2} \rightarrow$

b) $\frac{7}{10}, \frac{3}{5} \rightarrow \frac{7}{10}, \frac{3}{5} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$ e) $\frac{5}{18}, \frac{2}{3} \rightarrow$

c) $\frac{8}{15}, \frac{2}{3} \rightarrow$ f) $\frac{7}{45}, \frac{1}{3} \rightarrow$

1 Complete the table:

a	40	50	80	100	150	200	700	1000
1% of a	0.4							
5% of a	2							
10% of a	4							
60% of a	24							
140% of a	56							

2 Complete the table by following the given example.

a)

Fraction	$\frac{3}{100}$	$\frac{17}{100}$	$\frac{139}{100}$	$\frac{107}{100}$	$\frac{3}{20} = \frac{15}{100}$	$\frac{27}{25}$	$\frac{17}{50}$	$\frac{3}{5}$
%	3%							

b)

%	3%	7%	25%	70%	60%	22%	275%	37.5%
Common Fraction	$\frac{3}{100}$							
Decimal Fraction	0.03							

3 Find x if:



a) 12% of 2,500 = x ,22% of 2,500 = x ,42% of 150 = x ;

b) 9% of x = 54,6% of x = 480,25% of x = 24;

c) $x\%$ of 320 = 48, $x\%$ of 1,200 = 900, $x\%$ of 80 = 20.8.