

Zvezdelina Stankova, Sava Grozdev, Plamen Paskalev,  
Maya Alashka, Raina Alashka

# **BULGARIAN MATH 7** **OR 10A** **PROBLEMS COLLECTION** **AND WORKBOOK**

**adapted for U.S. High Schools**  
**used at UC Berkeley programs as college preparation**

Co-Authored, Translated, and Adapted by  
**Zvezdelina Stankova**  
Founder of the Berkeley Math Circle

ArchiMath  
ArchiMath Publishing LLC



**BERKELEY**  
**MATH CIRCLE**

© Publishing House “Archimedes 2” Ltd., 2025

© Zvezdelina Stankova, Ph. D., Founder of the Berkeley Math Circle and Math Taught the Right Way, Professor of Teaching Mathematics at University of California at Berkeley, CA 94720 – author, translator, adapter, editor, graphic designer, 2025

© Maya Alashka, Raina Alashka – authors, 2025

© Sava Grozdev – author, 2025

© Berkeley Math Circle Logo used on cover with per-mission from Berkeley Math Circle

© Emil Hristov – cover designer, 2025

Copyeditors: Adarsh Ganeshram, Allison Lew, Annie Yao, Anushka Mangal, Avery Lo, Claire Foster, Claire Liu, Daniella Averill, Erica Pan, Gavin Lai, Harish Premkumar, Ishwar Suriyaprakash, Jackson Gish, Jeffery Ren, Kalina Atanassov, Leena Wang, Leila Maboudian, Litong Deng, Lubo Zhou, Meagan Bald, Megan Gammill, Nathan Tran, Olivia Lee, Om Kannan, Shixuan Cao, Tobin Sudo, Zachary Smith, Zara Wang, UC Berkeley.

ISBN: 978-954-779-363-7

Printed by Alliance print – Sofia, Bulgaria

*These worksheets come directly from the 10A textbook. They consist of the examples from each lesson that have space for students' solutions and can be used for directed individual or group work in class.*

*The worksheets are to be used in tandem with the textbook. In addition to completing these worksheets, the full solutions, discussions, and remarks in the textbook should be carefully read and understood by the student.*

## CONTENTS

### Chapter 1. Initial Review

|                                             |    |
|---------------------------------------------|----|
| 1. Review Test 1 with Solutions .....       | 4  |
| 2. Review Test 2 with Solutions .....       | 5  |
| 3. Review Test 3 with Solutions .....       | 7  |
| 4. Review Test 4 with Solutions .....       | 8  |
| 5A. Review Test 5 with Solutions .....      | 10 |
| 7. How Mathematicians Write Solutions ..... | 11 |
| 8. How to Study for and to Take Tests ..... | 11 |

### Chapter 2. Power and Logarithm

|                                                           |    |
|-----------------------------------------------------------|----|
| 9. Third Root. Properties .....                           | 12 |
| 10. $n^{\text{th}}$ Root. Properties .....                | 13 |
| 11. Operating on Irrational Expressions .....             | 15 |
| 12. Working w/ Irrational Expressions. Exercises .....    | 16 |
| 13. Function and its Graph .....                          | 18 |
| 14. Graph of the Function $\sqrt{x}$ .....                | 20 |
| 15. Graphs of the Functions $x^3$ and $\sqrt[3]{x}$ ..... | 22 |
| 16. Power w/ an Integer Exponent. Applications .....      | 23 |
| 17. Power w/ a Rational Exponent. Properties .....        | 24 |
| 18. Power w/ a Rational Exponent. Exercises .....         | 25 |
| 19. Power and Exponential Functions. Graphs .....         | 27 |
| 20. Logarithm. Advanced Properties .....                  | 28 |
| 21. Logarithm. Exercises on Domains .....                 | 29 |
| 22. Logarithm. Comparing Logarithms .....                 | 30 |
| 23. Logarithmic Function and Its Graph .....              | 31 |
| 24. Logarithm of Product, Quotient, Power, Root .....     | 33 |
| 25. Change of Logarithmic Base. Exercises .....           | 34 |
| 26. Summary of "Power and Logarithm" .....                | 36 |

### Chapter 3. Stereometry (3d Geometry)

|                                                   |    |
|---------------------------------------------------|----|
| 28. Lines and Planes in Space. Angles .....       | 38 |
| 29. Relative Position of a Line and a Plane ..... | 39 |
| 30. Orthogonal Projections. Angles .....          | 41 |
| 31. Relative Position of Two Planes. Angles ..... | 42 |
| 32. Right Prism .....                             | 44 |
| 33. Pyramid .....                                 | 45 |
| 34. Pyramids with Equal Elements. Exercises ..... | 47 |
| 35. Right Circular Cylinder .....                 | 49 |
| 36. Right Circular Cone .....                     | 51 |
| 37. Sphere and Ball .....                         | 53 |

|                                                                       |    |
|-----------------------------------------------------------------------|----|
| 38. Summary of "Stereometry," Part I .....                            | 55 |
| Axiomatic Foundation. $\times$ , $\parallel$ , $\perp$ Lines & Planes |    |
| Proofs vs. "Obvious facts." Angles                                    |    |
| Drawing 3d-Objects on Paper Realistically                             |    |
| Properties of Any Projection                                          |    |
| 39. Summary of "Stereometry," Part II .....                           | 56 |
| $\perp$ , $\parallel$ Lines and Planes. Areas of Projections          |    |
| Orthogonal Transversal of Skew Lines                                  |    |
| Solids. Surface Areas and Volumes                                     |    |
| 40. Summary of "Stereometry," Part III .....                          | 57 |
| Incircles and Circumcircles in the Plane                              |    |
| Insphere and Circumsphere in Space                                    |    |

### Chapter 4. Algebra on Polynomials

|                                                     |    |
|-----------------------------------------------------|----|
| 42. Polynomials in One Variable.                    |    |
| Equal Polynomials and Zeros .....                   | 59 |
| 43. Operations on Polynomials.                      |    |
| Polynomial "Long" Division .....                    | 60 |
| 44. Rational Roots of an Equation                   |    |
| with Rational Coefficients .....                    | 62 |
| 45. Horner's Scheme, Part I                         |    |
| Applications to Polynomials .....                   | 64 |
| 46. Horner's Scheme, Part II.                       |    |
| Applications to Numerical Systems .....             | 66 |
| 47. Solving Some Equations of Higher Degree .....   | 68 |
| 48. Reciprocal Equations .....                      | 70 |
| 49. Symmetric Polynomials of Two Variables, I ..... | 73 |
| 50. Symmetric Polynomials of the Roots              |    |
| of a Quadratic Equation, II .....                   | 75 |
| 51. Summary and Extensions, I .....                 | 78 |
| Roots and Reciprocal Polynomials                    |    |
| Factoring Mirror Polynomials                        |    |
| 52. Summary and Extensions, II .....                | 79 |
| Symmetry and Factorization                          |    |
| Solving Homogeneous Equations                       |    |
| Ultimate Factorization and Irreducibility           |    |
| 53. Summary and Extensions, III .....               | 80 |
| Number Bases, Digits, and Divisibility              |    |
| Number Theory Mini-Test .....                       | 81 |

### Part II. Test Collection .....

**Example 1** Among six books, two are written by one author and the rest by different authors. We randomly arrange all books on the same bookshelf. The probability that the two books from the same author are next to each other is:

- A)  $\frac{2}{3}$ ;    B)  $\frac{1}{3}$ ;    C)  $\frac{1}{6}$ ;    D)  $\frac{1}{5}$ .

Solution:

**Example 2** The value of  $a$  for which the graphs of the functions  $f_1(x) = 7 - ax$ ,  $f_2(x) = 10 - 4x$ , and  $f_3(x) = 2x - 8$  pass through the same point is:

- A) 1;    B) 4;    C) 2;    D) 3.

Solution:

**Example 3** When  $\alpha = 30^\circ$ , the expression  $\frac{\cos(90^\circ - \alpha)}{\sin \alpha} \cdot \frac{\cos \alpha}{\sin(90^\circ - \alpha)} + \sqrt{3} \cot(90^\circ - \alpha)$  equals:

- A)  $\sqrt{3}$ ;    B) 3;    C) 2;    D) 4.

Solution:

**Example 4** The solutions to the system  $\begin{cases} x^2 + xy + y^2 = 37 \\ x - y = 1 \end{cases}$  are:

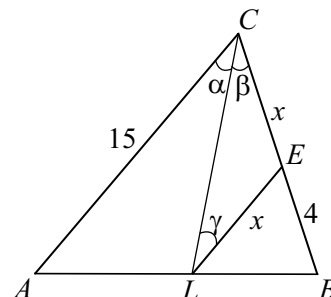
- A) (3, 4), (-4, -3);    B) (4, 3), (-3, -4);  
C) (4, 3), (3, 2);    D) (-4, -5), (3, 2).

Solution:

**Example 5** In  $\triangle ABC$ ,  $CL$  ( $L \in AB$ ) is an angle bisector. Consider the line through point  $L$ , parallel to  $AC$  and intersecting side  $BC$  in point  $E$ . If  $AC = 15$  cm and  $BE = 4$  cm, the ratio  $S_{\triangle LBE} : S_{\triangle ABC}$  is equal to:

- A)  $\frac{2}{3}$ ;    B)  $\frac{2}{5}$ ;    C)  $\frac{2}{9}$ ;    D)  $\frac{4}{25}$ .

Solution:



**Example 6** The sum of the smallest and the largest values of  $f(x) = -x^2 + 6x - 5$  on the interval  $[-1, 2]$  is:

- A) -12;    B) 3;    C) -9;    D) 9.

Solution:

## Review Test 1 with Solutions

L1

**Example 7** The domain of

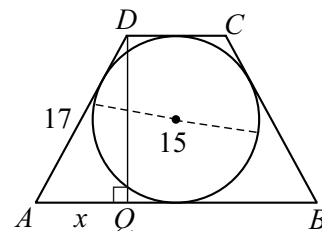
$$A = \sqrt{x^2 - 9} + \sqrt{5x - x^2} \text{ is:}$$

- A)  $x \in (-\infty, -3] \cup [3, +\infty)$ ;  
B)  $x \in [0, 5]$ ; C)  $x \in [3, 5]$ ;  
D)  $x \in [-3, 0]$ .

Solution:

**Example 8** About a circle of diameter 15 cm, we have circumscribed an isosceles trapezoid with a leg of 17 cm. Find the bases of the trapezoid.

Solution:



## Review Test 2 with Solutions

L2

**Example 1** The arithmetic mean of  $\{7, 4, 3, 9, 4, 7, 4, x\}$  is equal to 5.5. The median is:

- A) 5; B) 5.5; C) 6; D) 4.

Solution:

**Example 2** If we add the 3rd and 5th terms of an arithmetic progression, we will obtain 20, and if from the 7th term we subtract the sum of the 2nd and the 4th, we will obtain 5. The sum of the first 11 terms of the progression is:

- A) 167; B) 176; C) 175; D) 352.

Solution:

# PART II: 10A TEST COLLECTION

This 10A Test Collection (TC) contains 216 problems. It will save the teacher time in searching for more problems of various difficulty. 10A PC can be used for independent work by students, to assign homework and conduct tests, and for practical applications. The problems correspond to each chapter in 10A Textbook and are grouped in three levels.

## 1 Core Level: Mandatory

The *Core level problems* are for all students. They will strengthen and extend the basic knowledge and skills needed to understand the 10A material.

## 2 Advanced Level: Recommended

The *Advanced level problems* are for any student who wishes to master the 10A material and obtain excellent preparation for future math courses.

## 3 College Level: Elective



The *College level problems* are intended for the strongest students who wish to go beyond the 10A material and prepare for college courses. They are challenging and should be addressed only with proper support.

## Conventions, Abbreviations, References:

1. Unless otherwise specified, all variables are *real*, and  $x^y$  is considered only in the following three cases:
  - a)  $x > 0, y \in \mathbb{R}$ ;
  - b)  $x = 0, y > 0$ ; ( $0^0$  is *not* defined!)
  - c)  $x < 0, y = \frac{n}{m} \in \mathbb{Q}$  for  $n, m \in \mathbb{Z}, m$  odd (e.g.,  $x^5, x^{-4}, x^{1/3}$ ).
2. Diagrams may *not* be drawn to scale.
3.  $\lg x = \log_{10} x$  and  $\ln x = \log_e x$ .
4. All coins are *fair*, w/ equal probability of heads or tails.
5.  $V$ : volume;  $S$  and  $S_l$ : lateral and total surface areas.
6. *MUC*: Method of Undetermined Coefficients.
7. *EDA*: Euclid's Algorithm for finding the gcd of two polynomials, based on Euclidean division (cf. p. 98).
8.  $(x)^*$ : Part  $(x)$  is extra hard. Work on it with care.

# CONTENTS

## Chapter 1. Initial Review

|                 |    |
|-----------------|----|
| Tests 1-2 ..... | 84 |
| Tests 3-4 ..... | 85 |

### Challenge Topics

1. Square root cusp and regular hexagon
2. Parabola and regular hexagon
3. Parabola and right triangle
4. Parabola and equilateral triangle

## Chapter 2. Power and Logarithm

|                  |    |
|------------------|----|
| Tests 5-8 .....  | 86 |
| Tests 9-12 ..... | 87 |

### Challenge Topics

5. Arithmetic-harmonic inequality
6. Arithmetic-root mean square inequality
7. Arithmetic-geometric inequality
8. Arithmetic-geometric inequality
9. Parametric equation
10. Limit and rationalizing square roots
11. Properties of a special log-type function
12. Limit and rationalizing cube roots

## Chapter 3. Stereometry (3D Geometry)

|                   |    |
|-------------------|----|
| Tests 13-14 ..... | 88 |
| Tests 15-16 ..... | 89 |
| Tests 17-18 ..... | 90 |
| Tests 19-20 ..... | 91 |

## Challenge Topics

13. Inscribed cylinders of largest surface area
14. Externally tangent spheres
15. Cylinders of largest volume in a sphere
16. Balls, polar caps, and cylindrical hole
17. Venn diagrams of balls
18. Externally tangent spheres in a sphere
19. Inscribed cone of largest inradius
20. Inscribed cone of largest volume

## Chapter 4. Algebra on Polynomials

|                   |    |
|-------------------|----|
| Tests 21-22 ..... | 92 |
| Tests 23-24 ..... | 93 |
| Tests 25-26 ..... | 94 |
| Tests 27-28 ..... | 95 |

### Challenge Topics

21. Lagrange polynomials of degree 2
22. Translating polynomials to annihilate a term
23. Bombelli's guesses for complex cube roots
24. Conversions and digit predictions
25. Bases, #digits, and representations
26. Cardano-Tartaglia's formula in action
27. Gauss's lemma on primitive polynomials over  $\mathbb{Z}$
28. Eisenstein's criterion for irreducibility over  $\mathbb{Z}$

## Answer Key and Hints.....96

|                                  |    |
|----------------------------------|----|
| Power Means Epilogue .....       | 97 |
| Euclid's Algorithm for GCD ..... | 98 |

## Test 1. Initial Review

- ① A coin is tossed 10 times. The probability to get at least one head is:

A)  $\frac{1017}{1024}$ ; B)  $\frac{1019}{1024}$ ; C)  $\frac{1021}{1024}$ ; D)  $\frac{1023}{1024}$ .

- ② The value of  $\sin 390^\circ$  is:

A) 1; B)  $\frac{1}{2}$ ; C)  $\frac{\sqrt{2}}{2}$ ; D)  $\frac{\sqrt{2}}{3}$ .

- ③ Solve the inequality  $\frac{x}{x+3} \leq -2$ .

A)  $x \in (-\infty, -3)$ ; B)  $x \in (-3, -2)$ ;  
C)  $x \in (-3, -2]$ ; D)  $x \in [-2, +\infty)$ .

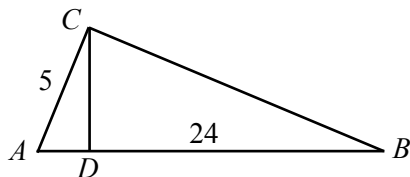
- ④ Suppose the pair  $(x, y)$  is a solution to

$$\begin{cases} 2x - y = 4 \\ (x-1)^2 + (y+2)^2 = 20. \end{cases}$$

The largest possible value of  $x + y$  is:

A) 4; B) 5; C) 6; D) 7.

- ⑤  $\triangle ABC$  has right angle  $C$  and altitude  $CD$ . The lengths of  $AC$  and  $BD$  are 5 and 24, respectively. How long is  $AD$ ?



- ⑥ Solve the equation  $x + 2 = \sqrt{10 - x}$ .

- ⑦ What is the value of  $a + b + c$ , where  $f(x)$  is the quadratic polynomial

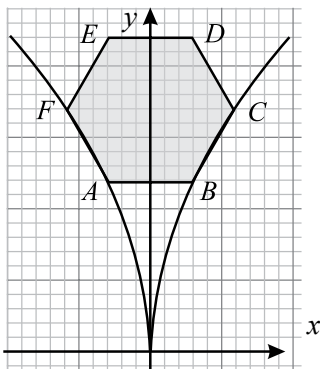
$$f(x) = ax^2 + bx + c$$

and  $f(x+1) = x^2 - x + 2f(6)$  for all  $x$ ?

### 8 (Challenging Cusp)



Vertices  $A, B, C, F$  of regular hexagon  $ABCDEF$  lie on the graph of  $f(x) = \sqrt{|x|}$ . Find the length of side  $AB$  if  $AB$  is parallel to axis  $Ox$  in the rectangular coordinate system  $Oxy$ .



## Test 2. Initial Review

- ① A coin is tossed 10 times. The probability to get between 4 and 6 heads, inclusive, is:

A)  $\frac{17}{32}$ ; B)  $\frac{19}{32}$ ; C)  $\frac{21}{32}$ ; D)  $\frac{23}{32}$ .

- ② The value of  $\cos 1110^\circ$  is:

A) 1; B)  $\frac{1}{2}$ ; C)  $\frac{\sqrt{3}}{2}$ ; D)  $\frac{\sqrt{3}}{3}$ .

- ③ Solve the inequality  $\frac{x+4}{x+1} > 2 - x$ .

A)  $x \in (-\infty, -1)$ ; B)  $x \in [-4, -1)$ ;  
C)  $x \in [-4, 2]$ ; D)  $x \in (-1, +\infty)$ .

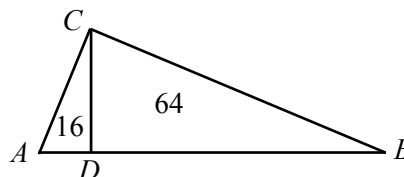
- ④ Suppose the pair  $(x, y)$  is a solution to

$$\begin{cases} x - y = 2 \\ 3xy - 2x + 2y = 20. \end{cases}$$

The smallest possible value of  $x + y$  is:

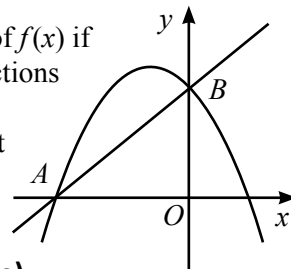
A) -8; B) -7; C) -6; D) -5.

- ⑤  $\triangle ABC$  has right angle  $C$  and altitude  $CD$ . The areas of  $\triangle ADC$  and  $\triangle DBC$  are 16 and 64, respectively. How long is  $AB$ ?



- ⑥ Solve the equation  $\sqrt{2x+3} = 6 - x$ .

- ⑦ Find the positive root of  $f(x)$  if the graphs of both functions  $f(x) = ax^2 + bx + c$  and  $g(x) = -ax + d$  intersect each of the two axes at  $A \in Ox$  and  $B \in Oy$ .



### 8 (Hexagonal Challenge)



Vertices  $A, B, C$ , and  $F$  of regular hexagon  $ABCDEF$  lie on the graph of the parabola  $f(x) = x^2$ . Find the length of side  $AB$  if  $AB$  is parallel to the axis  $Ox$  in the rectangular coordinate system  $Oxy$ .

